Damn Lies and Statistics
A Critique of Probability

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L'homme…est également incapable de voir le néant d'où il est tiré, et l'infini où il est englouti

—Pascal

1. Quality, Quantity, and Thresholds of Emergence

Blaise Pascal was the founder of modern probability theory and statistics, and understood probability better than any business actuary of his time. He knew he was cheating when he proposed his famous wager, since the probabilistic calculation of betting odds compares quantities, and cannot handle the infinity of eternal life and the nothingness of absence.[1] Worldly quantitative calculation could never capture the kind of qualitative difference in life he valued. What he might not have known was something that has only recently become clear: that a quantitative difference, if close enough to some great natural threshold or inherent and constitutive instability in the world, can trigger a qualitative difference. Today's market risk analysts, as Nassim Nicholas Taleb has pointed out in his book *The Black Swan*, are no better at detecting and evaluating the importance of such threshold-crossings, such inherent instability, than they were in Pascal's time.[2] The recent banking crisis was the result. Mark Twain's famous aphorism, quoted by Churchill—"There are lies, damn lies, and statistics"—has been proven true yet again.

That instability, that thresholdiness—the daemon that haunts all worldly calculation—has been more recently explored by three other remarkable Frenchmen—René Thom, Benoît Mandelbrot, and Rémy Lestienne, and

1 If you bet that there is an afterlife, you can't lose, since you will not be there to pay up if you're wrong. If you bet against it, you always lose: either you're right, but you're dead anyway and can't collect your winnings, or you're wrong, and must face an angry God.

2 *The Black Swan: Second Edition: The Impact of the Highly Improbable*. Random House, 2010.

a Belgian, Ilya Prigogine.[3] The instability is perhaps even more interesting than the nature of qualitativeness itself. It is the threshold between the quantitative and the qualitative *en soi*, the way that something can "amount to" something else.[4] In Mandelbrot's insight, a Peano space-filling curve, which is after all only a line, can "amount to" a plane, if a plane is defined as a two-dimensional space in which all the locations are occupied.[5] A frilly crocheted plane, a flower whose bell results from more growth of cells per open space than there is space for on the plane, can "amount to" a negative curvature and traces out a volume. Seven (but not five) H_2O molecules "amount to" water, with its constitutive wetness, flow, surface tension, ripples, bubbles, capillary action, drops, meniscus, and so on. It is only at the threshold of six that there emerges a sufficient numerical quorum of molecules to provide the right degrees of geometrical freedom, and thus exhibit the collective electric interrelation between them that generates these effects. A primitive light-sensitive spot on the head of an amphibian, with enough accumulation of transparent focusing tissue, "amounts to" an eye. A sufficiently large collection of self-organizing nerve cells "amounts to" a mind. An over-insured and over-secure real estate market can suddenly "amount to" an economy where people owe more money than there is in the world.

It is the existence of thresholds at all that is so remarkable. They are not confined either to the concrete or the abstract world. The traveling salesman problem can be expressed by a pure mathematical formalism, but the way that the mere addition of cities to his most efficient route so massively increases the difficulty of the calculation is the same both in mathematics and in physical space. And the degree of difficulty quite soon crosses the natural limit of the universe's computational capacity, even were it organized as the most efficient possible calculating engine. This difficulty, familiar to all who deal with limit theory and knots in mathematics, is itself a sensitive index of what we might call "thresholdiness." Indeed, the measure of difficulty, its tendency to increase exponentially with new variables in nonlinear systems, and its differential rates of increase in different circumstances, may be primitively constitutive of time itself, a fossil of the original instability, the emergence of temporality. The threshold (the present moment) of the past (all that might be known for certain) abuts upon the radical otherness of the unpredictable future. All we need for there to be a future at all is uncomputability.

New things emerge because thresholds await them. The thresholds are both necessitated by mathematical logic and encountered in the physical world, which remembers the limiting contracts that enabled its emergence. New things are like the bucket of water perched upon the proverbial door that will descend upon the unlucky victim of the practical joke when he pushes it far enough open. What are the conditions for that practical joke, what makes it possible, what rules would one need to have a universe free to invent radically new things without succumbing to mere inconsequential anarchy?

3 René Thom: *Structural Stability And Morphogenesis*. Westview Press, 1994. Benoît Mandelbrot: *The Fractal Geometry of Nature*. W. H. Freeman, 1982. René Lestienne: *The Creative Power of Chance*. University of Illinois Press, 1998. Ilya Prigogine: *The End of Certainty*. Free Press, 1997.

4 The English phrase "amount to" does not easily translate into French. "Revenir" implies a return to essential origins rather than the completion of a sufficient quantity to meet a goal; "décomposer" and "réduire" imply that the whole can be reduced to the parts. "Déboucher" might be a better translation.

5 A Peano curve is a fractal curve (not differentiable) that, although consisting of a simple line, fills the whole plane between determined x,y limits.

2. Digital and Analog: The Need for Granularity in a World of Thresholds

One of those conditions is that a universe capable of threshold-crossing, of emergence, cannot be perfectly analog, in the sense of being totally dependent on continuous variables. It cannot be resolvable into more and more minute gradients of quantity, and cannot be fully understood by the smooth bell-shaped curves of probability. The "thresholdy" universe must be granular, quantized, digital as opposed to analog, at some fundamental level. It must be made up of "pixels," so to speak, atoms in the old Greek sense of the term. (Contemporary particle physics now knows of much smaller pixels than the atoms, but in the Greek sense those smaller irreducible chunks—whether quarks or strings—are the new atoms).

"Time," said Heraclitus, "is a child playing a game of draughts; the kingship is in the hands of a child" (Fragment 52: K. Freeman translation), which I take to mean that the mutual prediction contest—the second-guessing that gives all games their suspense and thrill—is at the heart of the nature of time, its strange asymmetry between the past and the future. The Hindus, too, regard time as a *lila*, a game. And all games require the equivalent of distinct counters, turns, and players. A tennis ball is either in or out. A chess turn, a chess piece, a chess square are fundamental quanta, granules, of the game. Without turns, the players cannot synchronize enough to have a contest at all. Without individual players with distinct interests, neither prisoner in "Prisoner's Dilemma" could wish to rat out his accomplice. Electronic calculation is itself a useful game, using distinct ones and zeroes; all over the world, engineers are looking for ways of making smaller and smaller secure thresholds to hold and transfer bits of information. Even quantum computers only kick the problem of keeping the counters of the game distinct down to the quantum level. The "calc" in "calculation" is a Greek pebble or abacus-bead used in geometry and arithmetic, and also in children's board games of the times.

Paradoxically, it is only when we play with distinct pieces, and the defined rules that identify them, that the true mysteries and discoveries can happen: because it is only if the lines are sharp, and the definitions granular, that the fertile paradoxes can appear. Imaginary numbers—the way in which the number line gives birth to the Hamiltonian plane in response to the need for another orthogonal space to accommodate the imaginary numbers—could not come to be without the distinct plus and minus signs, the distinctness of both the natural numbers and the exponents that transform them.[6] Individuality in its most primitive sense is necessary for invention.

It does not matter if those counters and turns—the quanta and chronons of the world—are only relative to some particular feature of the universe, say sound or light or living cells. They can be fundamental and relative at the same time. No event can be shorter than the Planck time or happen in a smaller space than the Planck length; no sound for a human ear can be shorter than one twenty-thousandth of a second, 20 kHz being the highest pitch it can hear. No piece of light is smaller than the wavelength of its photons. No cytological activity can take place on a smaller level than a cell. No vote can be cast by less than one person, no sonnet recitation last less than about thirty seconds.

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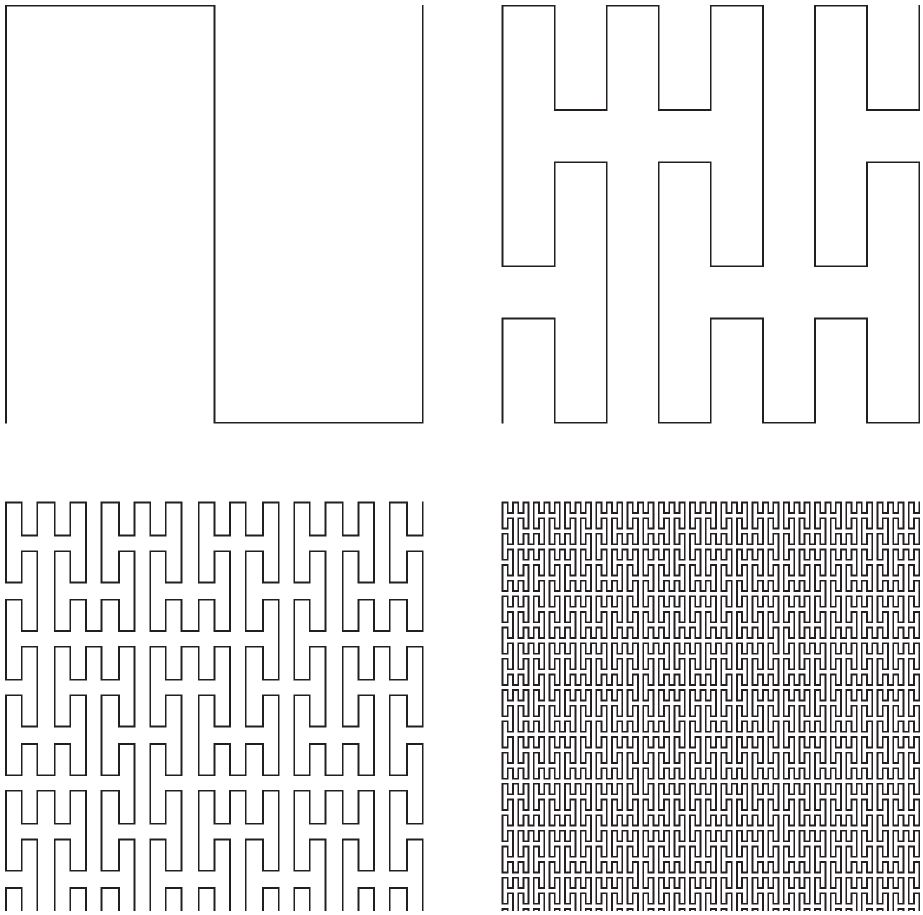
6 If at certain points in this essay it is difficult to tell whether the calculative process referred to is that of the analyst of the universe or the universe itself, the ambiguity is intentional. I take it that a.) whatever we have to calculate, the universe must itself have had to work out first; and b.) we are part of the universe anyway, and our calculation process is part of the universe's calculation process. Hamiltonians are both a human concept and a natural organizational structure.



A Peano space-filling curve in four steps. (Wikimedia/Creative Commons)

3. The Problems of an Analog Universe

Let us perform a thought experiment and imagine a purely analog probabilistic world. It is one of continuous gradients and variations in mixtures. Claude Shannon pointed out that information can only be transmitted, can only exist at all, if the magnitude of its departure from the default state of its medium, channel, or carrier-wave is enough to cross some threshold that distinguishes it from noise.[7]

The probabilistic/analog world is, so to speak, all noise. It cannot make explosions or compounds (as opposed to mixtures). It is all bell-curves, it has no cusps, no catastrophes. It has no states of matter; Gibbs' free energy law, which governs such phenomena as freezing, boiling, melting, evaporating, precipitating, condensing and so on, does not hold because there are no natural thresholds to cross. No new species, no new ecological niches, no new works of art could emerge into existence, crossing the boundary from the unimaginable to the possible.

7 Claude E. Shannon: *A Mathematical Theory of Communication*. University of Illinois Press, 1949 (reprinted 1998).

If everything merges smoothly into everything else, if everything gradually becomes everything else, there can be no game. Points could not exist, and thus could not cluster together to make lines. Lines could not stitch themselves into planes, planes could not rumple and frill themselves into volumes. Time could not have distinct beats, and thus length; it could not mount up and thus could not have a direction; it would be an eternal amorphous cloud of becoming. The change of phase among solid, liquid, gas, and plasma, between crystalline and amorphous, could not happen. Functional individualities make available the strategic back-and forth of feedback, the competition and cooperation among regimes of crystallization or polarization in a metastable melt, among rock anemones in the ocean, predators and prey in the steppes, stock investors in the market, or nations in global politics, that lead to emergent ecological niches, technologies, and polities. It is only by such interactions that things can "amount to" something other than themselves, that the whole can be greater than the parts, and that the crises, bouleversements and dénouements of evolution can be free to occur. Without distinct notes, there could be no music. Without distinct words, there could be no language. Without distinct lines, there could be no poetry.

This is not to say that the analog aspects of the universe—those that are quantifiable and divisible all the way down, and are subject to probabilistic expression and statistical analysis—are an illusion or unimportant or an obstacle to progress. Much of the universe, much of the time, is fairly accurately describable by approximations and averages, and we are fairly safe when we "round things off." Many varying conditions do indeed regress to the mean. Chi-square tests for goodness of fit are rightly persuasive. But the success of probability theory as a way of predicting events and describing states too complex to be tractable in terms of Newtonian determinism, and its reliable use in the thermodynamic understanding of gases, work, entropy and even in quantum mechanics, have led to an overestimation of the extent of probability's writ. Probabilistic mathematics can handle negative feedback, but not positive feedback when it crosses thresholds that define natural states. The resulting errors are especially glaring in evolutionary biology, the social sciences, public policy, and the arts and humanities.[8] The mutation that triggers the emergence of a new species, the assassination that triggers a world war, the dream that inspires a masterpiece, cannot fit a system of standard deviations. What makes a human being a human being is precisely what differentiates that human being from their demographic.

Autonomy literally means "the making of rules for ourselves that we obey"—and the paradox occurs when we ask whether we are obeying the rules when we make them up, and whether once we are obeying the rules we have made, we are still autonomous as when we made them up.

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8 Rémy Lestienne (personal communication) points out that Ludwig Boltzmann never believed that nature was analogic and thought that analogic physics was only a mathematical trick. Lestienne adds that Henri Bergson went so far as to identify true freedom of will as itself exclusively a moment of creative emergence—"les actes libres sont rares"—while most of our decisions remain physiologically determined.

4. Time as Difficulty

It is the continuous competition and coexistence of the analog and the digital, the probabilistic and the "thresholdy," that enables the curious open-ended creativity of the universe. There *is* an analog logic—it is the logic of the quantum computer, in which the yeses and noes of Aristotelian logic are replaced by superimposed statistical likelihoods harmonized with each other by entanglement. The mismatch between the two logics, and the continuous *difficult* calculation process that reconciles them in an *ad hoc* accommodation, is perhaps part of the constitution of time itself. The difficulty of solving difficult algorithms, like the calculation of factorials or the traveling salesman problem, is due directly to the nestedness of sub-calculations and sub-sub-calculations that must be solved before each step in the process.

Out of this recalcitrance emerges a primitive form of sequentiality, an asymmetry between the ease of, for instance, the simple multiplication of a set of numbers, and the exponentiating difficulty of the reverse, that is, the extraction of the factors of the large numbers that result.[9] Significantly, a quantum computer, clumsy at classical computation, can in theory solve factorial problems with ease, being unburdened by temporal order, while a classical computer, struggling with scheduling problems, is quickly stymied when the number to be factored gets too large. We might speculate that each new emergent entity in the world is the latest attempt at solving the paradox of the coexistence of both kinds of computation.

5. Inventing a Free (and Therefore Survivable) Universe

If one were tasked to invent a survivable universe, it would be hard to avoid the singularly ingenious solution to the problem that we find in this one. A survivable universe is one that generates a new moment every moment, a new moment that reliably encodes the previous moment but is not encoded by it. It must be retrodictable but not predictable: it must be genuinely branchy as we go forward in time, and genuinely single when we look back at it. Such a universe must be continuous in both space and time (or it would not be one but many universes). But the continuity should not be in the trivial sense that a point is a very small circle or that something is continuous with itself because it is identical to itself, or the musical note C4 is in harmony with C5. It must be continuous but asymmetrical with respect to space and time.

The solution seems to be to make the basic constituents of the universe quantized—digital; but make the logic by which they interact with each other and with themselves probabilistic—analog. Then let its logic transform to digital once a certain size and duration threshold (the

9 Michael Heller (see my review of his *Creative Tension: Essays on Science and Religion*, Templeton Foundation Press, 2003 in *Kronoscope*, 11, 2011, p. 167) argues that noncommutativity, i.e. an asymmetry in logical order in respect of the identity relation, is all that is needed to get a universe. From my review: "For Heller, the essential issue is how timeless mathematics—which, he argues, miraculously does truly describe the real world—actualizes itself in matter and time. He suggests interestingly that the mathematics of quantum theory is noncommutative. That is, unlike commutative mathematics in which 3×7 is the same as 7×3, the noncommutative mathematics of the first moment of the Big Bang and of any space in the present universe smaller than the Planck length dictates a difference in the state of a system according to the order in which a mathematical or logical operation is performed. That is, if 3×7 is not the same as 7×3, the difference is the fundamental unit of space and time. Given such units, a whole universe can evolve without outside assistance through the now familiar processes of selection, self-organization, and emergence in nonlinear dynamical systems…"

quantum/classical divide) is passed. The fine-grain logic of the universe is fuzzy; the coarse-grained logic is hard-edged and granular. The basic quanta of our hypothetical universe, its atomic pixels, work together by analog probabilistic rules of combination—quantum logic—rules that are different from those of its coarse-grained logic, which is classical, Aristotelian, digital. Make the world out of very tiny indivisible pebbles, or calculi, and make them chunk only at certain specific thresholds—but make the fine-grain logic, by which their interactions and their chunkings happen at the most fundamental level, probabilistic and always analog and curvy, branchy and inexact at some level of magnification.

Then let a more digital logic emerge in the interactions of the chunks that result. In large numbers those chunks themselves will still exhibit collective statistical properties, but only up to the point where some threshold of overcrowding suddenly appears, such as when enough molecules exist in a space to constitute a gas, with emergent collective properties like pressure and temperature. But the really ingenious twist is that those chunks must compete for existence; and their existence, their individuation, is assured only by their internal process being so difficult to predict that they cannot be absorbed by some more complex and unpredictable chunk or system of chunks, with its own prepared niche and procedure for modeling and incorporating subordinate chunks. They are game-players already, unconsciously outthinking each other. This evolutionary process produces structures that act *in anticipation of each others' actions*, creating a new indeterminacy of strategic competition and cooperation.

The final result of the struggle was the emergence of very large and complex individual (digital) organisms such as ourselves, that possess the emergent property of freedom, an instantiation of the paradox of autonomy. Autonomy literally means "the making of rules for ourselves that we obey"—and the paradox occurs when we ask whether we are obeying the rules when we make them up, and whether once we are obeying the rules we have made, we are still autonomous as when we made them up. Are we constrained to only make rules that are amendable, like the U.S. Constitution? Is such a constraint itself amendable, as when we bind ourselves to a solemn promise? Such a promise may be our freest moral act—a choice not only of what we do, but who and what we are. The match between such hypothetical issues—predicted by the tension between the digital and the analog, the probabilistic and the "thresholdy"—and our actual experience, is quite striking.

A quantum computer, clumsy at classical computation, can in theory solve factorial problems with ease, being unburdened by temporal order, while a classical computer, struggling with scheduling problems, is quickly stymied when the number to be factored gets too large.

6. Better Models for the Emergent Properties of Time

Neither inductive reasoning, which is generally quantitative, probabilistic, and analog (and characteristic of British empiricism) nor deductive reasoning, which is generally qualitative, "thresholdy," and digital (and characteristic of French rationalism) can by themselves account for the emergent properties of time. What kinds of models and strategies might work better?

The first recommendation might be the avoidance of certain kinds of errors. As we have seen in our hypothetical universe, to construct the kind of ordered unpredictability we need there must be a curious set of reversals, perhaps heralded at the beginning by the wave-particle dualism of the elementary structures of nature—a kind of sawing back and forth between apparent constraints upon free play, and apparent dissolution of all law and necessity. The digital graininess of the basic pixels of the world would seem to imply a fixed deterministic order, such as that suggested by Laplace's famous calculator, which, programmed with the position, momentum, and vector of every particle in the universe, would be able to accurately predict all events. That set of predictions would constitute an eternal and instantaneous singularity, with no need to work them out in time, and would not correspond to the messy and unpredictable universe we have actually got.

This first error, however, is compounded by a second one: that the observed unpredictability or randomness of individual quantum events is in itself a true escape from the chains of determinism. In different ways Erwin Schrödinger, Hugh Everett III, and Roger Penrose have entertained and struggled with this proposition; but randomness is surely even further away from the observed autonomy of the world's inhabitants than is the Calvinist or determinist notion of Fate or necessity.[10] The "degrees of freedom" found in the statistical logic of quantum mechanics are not free in the sense that a fish or a philosopher are free. And the Many Worlds hypothesis, often used to reconcile determinism with quantum randomness, cries out for Occam's razor—any other explanation must be superior by definition. The randomness of the behavior of the universe at its most fundamental level can provide choices, but it cannot choose. In an analog universe of superpositions, an infinite number of shades of color are available, but choosers of one in particular do not exist.

The second reversal is the way that digital individuality makes possible the emergence of strict rules and symmetries—an apparently deterministic feature. But rules, symmetries and constraints create thresholds, and thresholds make possible the crossing of thresholds and the emergence of new structures and functions. The evolutionary competition of such structures, with their new functions, produces entities that begin to show the characteristic features of true freedom, the ability to strategically contest or choose from the various determinisms on offer.

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10 Erwin Schrödinger: "Science and Humanism" in '*Nature and the Greeks*' and '*Science and Humanism*'. Cambridge University Press, 1996.
Simon Saunders, Jon Barrett, Adrian Kent, David Wallace, eds.: *Many Worlds? Everett, Quantum Theory, and Reality*. Oxford University Press, 2010.
Roger Penrose: *The Emperor's New Mind*. Oxford University Press, 2002.

7. Promising Directions for Further Inquiry

Thus one direction that one might take philosophically, so as to be able to transcend the mistakes of both deterministic and probabilistic thinking, might be in the direction of American pragmatism in the tradition of C. S. Peirce, William James, and John Dewey, and French evolutionary philosophy in the tradition of Henri Bergson and Teilhard de Chardin. As poetic metaphors, their conception of progress (with its implicitly tragic undertones) might serve as a good corrective to the simplemindedness of reductionistic positivism on one hand, and deconstructionist arbitrariness on the other.[11]

The view of emergence taken by some implies that information (and later knowledge, when knowers evolved) is immanent in the universe, or let us say *increasingly immanent,* as structures of matter increasingly acquire the ability to model and predict each others' behavior.[12] Thus we should be looking for ways to explore the curious correspondences between information theory on one hand and thermodynamics on the other. The way in which living organisms use the flow of increasing entropy as a collection device to acquire useful models of the future and fuel for behaviors and structures that anticipate it, is very suggestive. In what sense and circumstances can the increase of thermal disorder be also a potential increase in meaning, of informational order? In the discipline of economics there have been a series of breakthroughs, from a mechanistic, probabilistic, statistical, econometric approach, to an understanding of rational expectations, to a recognition of biopsychological biases on rationality, and to a new understanding of the intentionality of crowds. What everything and everybody knows about everything and everybody else is not an epiphenomenon of the world but a force locally more powerful than gravity or electricity. The emergent tail wags the causational dog.

Chaos theory and complexity theory have supplied us with mathematics, models, and images for reconceptualizing the world in an emergentist perspective. Much work has already been done in this direction: to quote from my own preface to *Chaos, Complexity, and Sociology: Myths, Models, and Theories*:

> *What the new science has done in effect is to place within our grasp a set of very powerful intellectual tools—concepts to think with. We can use them well or badly, but they are free of many of the limitations of our traditional armory. With them we can dissolve old procrustean oppositions—between the ordered and the random, for instance—and in the process reinstate useful old ideas like freedom. New concepts, such as emergence, become thinkable, and new methods, such as nonlinear computer modeling, suggest themselves as legitimate modes of study. I have divided these new conceptual tools into six categories: a new view of cause and prediction, a richer understanding of feedback and iteration, a revolution in the idea of time, an anthology of new recognizable structures and shapes, the idea of the attractor as a way of dissolving old dualisms, and the technique of modeling.*[13]

Game theory offers us powerful experimental tools, conceptual tools, and mathematical techniques for analyzing

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11 As highlighted by the fact that Alan Sokal's hoax essay "Transgressing the Boundaries: Towards a Transformative Hermeneutics of Quantum Gravity" was published by the postmodern journal *Social Text*.

12 Spinoza, Rémy Lestienne points out, came to a similar conclusion, that the physical and mental worlds were one and the same.

13 Raymond A. Eve, Sara Horsfall, Mary A. Lee: *Chaos, Complexity, and Sociology: Myths, Models, and Theories*, Sage Publications, 1997, pp. xi-xxvii.

strategic behavior—and a recognition that such behavior extends deep into the evolutionary roots of life on earth, and perhaps further still.

For a grand philosophical overview we need go no further than that of J.T Fraser's monumental conspectus of the temporal levels of the world, with its splendid account of the mechanisms by which new levels emerge from the old.

And if we are troubled by the reflection that as rational thinking beings we can have no intuitive understanding of the process of emergence, or that it is pointless to try to analyze the inherently unpredictable, there is a talent that we possess that may console us. I propose that the human aesthetic sense is precisely the capacity that an advanced animal with brain tissue to spare might develop to both guess and contribute to the course of emergence as it occurs around us on both the large and the small scale. What we find beautiful may be said to be what is about to emerge, what is emerging, what reveals its emergence. Art and poetry are the way that we use the hugely complex, multiply iterative, and astonishingly adaptive tissue of our nervous system to continue the invention of the world. A new work of art, whether a sonata, a fresco, a sonnet, or a lovingly-raised child, is the most improbable thing in the world, and the most valuable for that reason. A

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